Dual Nature of matter and Radiation

Einsteins photo electric equation is

$$h\nu = \frac{1}{2}mv_m^2 + \phi_0.$$

or $$\frac{1}{2}mv_m^2 = h\nu - \phi_0.$$ where $\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$.

$$\frac{1}{2}mv_m^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

* It is in accordance with Law of conservation of energy.
* Energy of the incident photon ($h\nu$) = energy spent in taking the $\bar{e}$ just out of the photosensitive surface (ϕ_0) + K.E of the photo $\bar{e}$
* The photo $\bar{e}$s are ejected with velocities ranging from 0 to v_m. So they are ejected with different kinetic energies even when the incident photons have same energy. This happens because all the $\bar{e}$s do not exist in the surface layer. Those coming from below the layer lose more energy in getting them free.
* The workfunction is the minimum amount of energy spent in taking the photo $\bar{e}$ out of the metallic surface. & $\phi_0 = h\nu_0$ ($\nu_0 \rightarrow$ threshold frequency for a metal).
* If the collector of the photocell is given negative potential, the photo $\bar{e}$s moving towards it will be retarded. The negative potential of the collector, which is just enough to stop the photo $\bar{e}$ with max. velocity, is called the stopping potential. In such a case,

$$\frac{1}{2}mv_m^2 = eV_0$$ where $V_0 \rightarrow$ stopping potential.

∴ photoelectric equation may be re written as
$$h\nu = \phi_0 + eV_0.$$

* The photoelectric emission is instantaneous, ~~spontaneous~~ i.e the photo $\bar{e}$ is ejected as soon as the photon is absorbed.

* photoelectric effect conforms the particle or quantum nature of light.

(i) photoelectric effect occurs, when the energy of the incident photon is of the order of workfunction ie $h\nu \approx \phi_0$.

(ii) The $\bar{e}$ ejected in photoelectric effect completely absorbs the incident photon.

(iii) All of the incident photon do not cause photoelectric emission.

* In photoelectric effect, the energy of the incident photon is of the order of binding energy of the electron ie electron is assumed to be bound..

* For alkali metals, the threshold frequency lies in the visible region. For zinc, it lies in the UV region. IR radiations cannot eject photo $\bar{e}$s while x-rays does it always.

[In compton effect, a part of the photon energy is absorbed by the electron. The max. energy that the photon can lose in compton effect is given by
$$E = \tfrac{1}{2} m_0 c^2 \,.]$$

Comparison of wave theory and photon theory for photoelectric phenomenon

* According to wave theory, when a wavefront strikes a metal surface, the free $\bar{e}$s at the surface absorb the radiant energy continuously,

* Greater the intensity, greater are the amplitudes of the wave. This indicates that higher intensity wave should liberate photo ē's with greater K.E.

* A light wave of sufficient intensity ejects ē's, whatever may be its frequency & energy & each ē intercepts an insignificantly some amount of energy and hence would require a finite time to escape from the metal surface. [Energy of light wave is smoothly & evenly distributed over a wavefront]

* No matter what the frequency of incident radiation is, a light wave of sufficient intensity (over a sufficient time) should be able to impart energy required to eject ē's from the metal surface which ~~expl~~ fails to explain the existence of threshold frequency.

* where as, according to photoelectric effect, max. K.E of emitted photo ē's is independent of the intensity of incident radiation and is dependent on frequency of incident radiation

* High energy (higher frequency as $E = h\nu$) radiations of very low intensity can emit photo ē's, whereas higher intensity but low energy (below threshold energy) cannot emit photo ē's.

* Emission of photo ē's is instantaneous (within 10^{-9} sec).

Main points of the Einstein theory of photoelectric effect are :-

(i) photoelectric emission is the result of interaction of 2 particles — one a photon of incident radiation and the other an $\bar{e}$ of photosensitive metal.

(ii) The free $\bar{e}$s are bound within the ~~metal~~ metal due to restraining forces on the surface. The min. energy required to liberate an $\bar{e}$ from the metal surface is called workfunction ϕ_0 of the metal.

(iii) The energy ($h\nu$) of the incident photon is used in two ways

(a) a part of it is used in relieving the $\bar{e}$ from the metal surface, which is equal to workfunction ϕ_0 of the metal.

b) remaining energy is used in imparting K.E to the ejected electron.

(iv) Very few ($< 1\%$) photons, whose energies are greater than ϕ_0 are capable of ejecting the photo $\bar{e}$s.

$$h\nu = \frac{1}{2} m v^2_{max} + \phi_0$$

$$K_{max} = \frac{1}{2} m v^2_{max} = h\nu - \phi_0 \quad \text{——— (1)}$$

If the incident photon is of threshold frequency, ν_0, then its energy $h\nu_0$ is just enough to free the $\bar{e}$ from metal surface & does not give it any K.E.

So $h\nu_0 = \phi_0$.

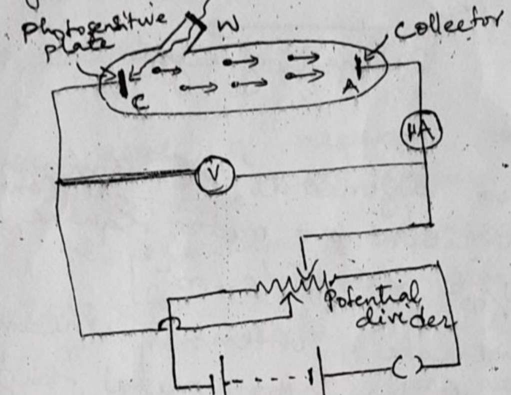
$$\therefore \quad K_{max} = \frac{1}{2} m v^2_{max} = h\nu - h\nu_0 = h(\nu - \nu_0) \quad \text{— (2)}$$

Eqns (1) and (2) are called Einstein's photoelectric equations.

Dual Nature of matter & Radiation

Experimental study of photoelectric effect

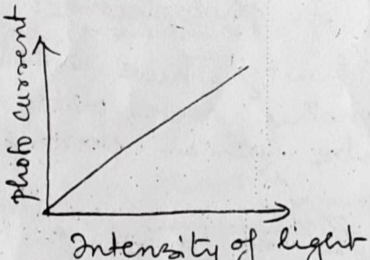
Figure shows the experimental arrangement used for the study of photoelectric effect. It consists of an evacuated glass/quartz tube which has a photosensitive plate C and another metal plate A. A quartz window W is sealed on the glass tube which permits radiations to fall on plate C.



The photo ēs emitted by the cathode C and collected by plate A, (anode / collector). The 2 electrodes are connected to a high tension battery thru' a potential divider arrangement. C is connected to the midpt. of potential divided but potential of A is made +ve or -ve w.r.t plate C. Microammeter measure the photoelectric current and voltmeter measures the p.d. applied b/w 2 electrodes.

1) Effect of intensity on photoelectric current:

For a radiation of fixed frequency falling on plate C and constant p.d. b/w the electrodes, the photoelectric current is found to increase linearly with intensity of incident radiation. This implies that (no. of photo ēs emitted per second is proportional to the intensity of incident radiation which) is shown below graphically.



$E/\Delta t$ or P/A

Explanation:

[Intensity is defined as the amount of light / photon energy incident per metre square (or per unit Area) per second.
SI unit is W/m^2 or $J/s\text{-}m^2$]

The increase of intensity means increase in no. of photons striking the metal surface / unit time. As each photon ejects only one e^-, no. of ejected e^-'s increases with increase in intensity of radiations.

[Energy of each photon, $E = h\nu = \frac{hc}{\lambda}$

Let total energy emitted per second by a source be P, i.e. Power of the source, then

~~no. of~~ $P = nh\nu = nE$

or $n = \frac{P}{h\nu} = \frac{P\lambda}{hc}$ gives no. of photons emitted / second by a source of P watts]

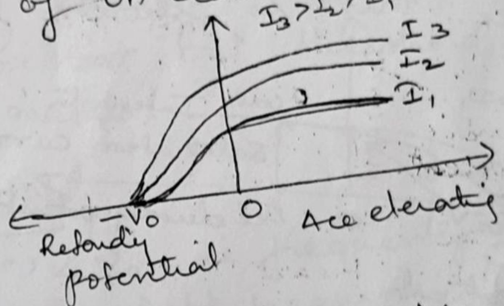
2) **Effect of Potential**: If we keep the intensity I, ~~fixed~~ and frequency of incident radiation fixed and increase the +ve potential (accelerating potential) on plate A gradually, it is found that photo current increases with increase in accelerating potential till a stage is reached when it becomes maximum & hence does not increase with further increase in accelerating potential. At this stage, all the photo e^-'s emitted by C is collected by A.

②

If we repeat the experiment with incident radiation I_2, I_3 etc such that $I_3 > I_2 > I_1$, it is observed that values of max. current also increases in proportion to the increase in intensity.

This (max. value of photo current, which corresponds to a particular value of intensity of incident radiation is called saturation current)

Now if we apply a negative potential on A w.r.t plate C, & increase its value gradually, it is seen that photocurrent decreases rapidly until it become zero for a certain value of negative potential on plate A. The value of retarding potential at which photoelectric current becomes zero is called cutoff or stopping potential for a given frequency of incident radiation.



At the stopping potential V_0, when no photo ē's are emitted, (ie it is just sufficient to stop the fastest ē as the velocities of photo ē's ranges from 0 to v_{max}), the work done by stopping potential on the fastest ē must be equal to its kinetic energy ie

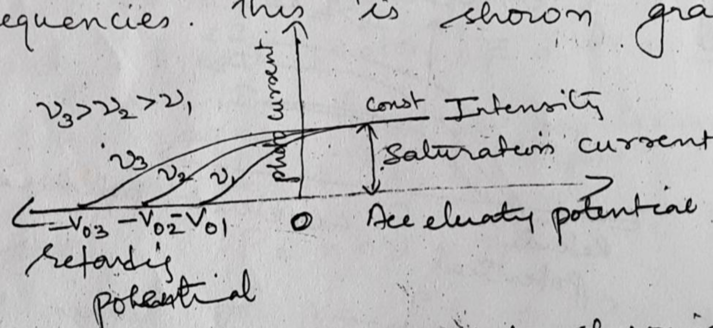
$$K_{max} = \frac{1}{2}mv^2_{max} = eV_0$$

e → charge on an ē
m → mass of an ē
v_{max} → max velocity

Explanation:-

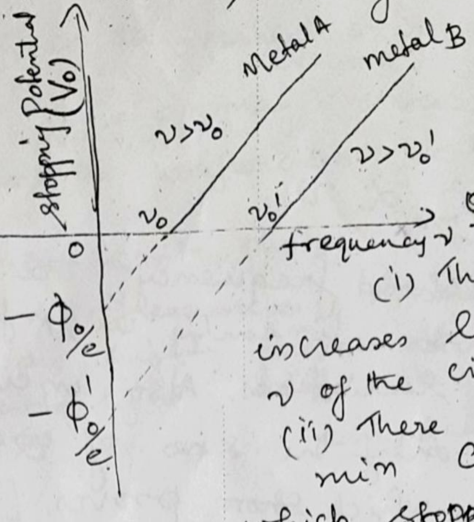
3) Effect of frequency of incident radiation on stopping potential

The frequency of incident radiation is varied keeping the intensity of radiation fixed and the above experiment is repeated ie the potential on plate A is made positive/negative as desired ie the potential is made gradually zero (from +ve) after attaining saturation current and then increased in negative direction till stopping potential is reached. The expt is repeated with radiations of different frequencies. This is shown graphically as



It is seen that value of stopping potential increases with the frequency of incident radiation. ie for frequencies $\nu_3 > \nu_2 > \nu_1$, stopping potential vary as $V_{03} > V_{02} > V_{01}$. This implies that greater the frequency of radiation, greater is the max K.E of the photo e^-s and hence greater is the retarding potential required to stop such e^-s completely.

If we plot a graph b/w frequency of incident radiation and the corresponding stopping potential for different metals, we get a graph as shown below.



OBSERVATIONS

(i) The stopping potential increases linearly with frequency ν of the incident radiation

(ii) There exists a certain min cutoff frequency for which stopping potential is zero.

The min. value of frequency of radiation below which photo electric emission ~~~~ does not occur is called Threshold frequency.

(iii) For 2 diff. metals A & B, These graphs are parallel and straight-lines i.e They ~~~~ have same slope ~~~~ but threshold frequencies are diff. for 2 metals.

EXPLANATION

If $\nu < \nu_0$, i.e frequency of incident radiation is less than threshold frequency, according to Einstein's photoelectric equation K.E of photo electrons become negative i.e

$$K_{max} = h(\nu - \nu_0)$$

which implies that photo $\bar{e}$ emission does not occur below threshold frequency as K.E cannot be $-ve$.

If $\nu > \nu_0$, then

$$K_{max} = \frac{1}{2} m v_{max}^2 \propto \nu$$

ie above threshold frequency, the max. K.E of the $\bar{e}$ increases linearly with frequency ν of the incident radiation. Also increase in intensity increase only the no. of incident photons & not their energy which proves that max K.E of photo $\bar{e}$'s is independent of intensity of radiations.

Determination of planck's constant & workfunction

According to Einstein's photo electric equation, the max K.E is given by

$$K_{max} = h\nu - h\nu_0$$

If V_0 is the stopping potential, then

$$K_{max} = eV_0$$

$$\therefore eV_0 = h\nu - h\nu_0$$

$$V_0 = \frac{h}{e}\nu - \frac{h}{e}\nu_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

This equation is of the form

$$y = mx + c$$

where $m \rightarrow$ slope of graph is h/e

ie $m = \tan\theta = \frac{h}{e}$

$\therefore h = m \times e$ ie slope of the graph $\times e$.

Thus planck's Const. 'h' can be determined.

The intercept on the vertical axis (y-axis) $= -\phi_0/e$

$\therefore \phi_0 = e \times$ mag. of intercept on y-axis

which gives the workfunction for the metal.

[Diff. substances emit photoe⁻s only when exposed to radiations of diff. frequencies. Alkali metals like Li, Na, K, Cs & Rb are highly photosensitive which emit e⁻s with even visible light. Metals like Zn, Cd, Mg, Al etc respond only to U.V. X-rays can eject e⁻s even from heavy metals].

* photo emissive cells, photo cells etc work on the principle of photoelectric emission.
* photocells are used in cinematography, counting devices, Burglar alarm, fire alarm, cameras, street light systems etc. A photocell converts a change in intensity of radiation into a change in photo current.

<u>Photon picture of light/electromagnetic Radiation</u>

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons

(ii) Each photon has energy $(E = h\nu)$ and momentum $(p = h\nu/c)$ and speed c, the speed of light

(iii) All photons of light of a particular frequency ν and wavelength λ, have same energy and momentum whatever be its intensity ie photon energy is independent of its intensity. Energy does not change when it moves from one medium to another.

(iv) They are electrically neutral ie not deflected by electric and mag. fields.

(v) In a photon-particle collision (elastic), the total energy & total momentum are conserved. However total ~~travel in straight line~~.

no. of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

[(vi) The rest mass (m_o) of the photon is zero which means energy of the photon is completely kinetic

Momentum of the photon is

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

Mass of moving photon is given by

$$m = \frac{h\nu}{c^2} = \frac{h}{c\lambda} \text{]}$$

$\frac{1}{\lambda} = \frac{\nu}{c}$, $c = \lambda\nu$

Dual Nature of Radiation

In 1924, the ~~French~~ French physicist Louis Victor De Brogie put forward the hypothesis that material particles in motion exhibit wave-like properties based on assumptions that

(i) The 2 physical quantities which governs all forms of universe are matter and radiation, (or mass & energy)

~~The~~ Einsteins mass-energy equivalence is

$$E = mc^2$$

This specifies that there should be a mutual symmetry between matter and radiation.

(ii) Nature loves symmetry. Since radiation has dual nature [ie interference, polarisation, diffraction etc shows that light has wave nature while photoelectric effect, compton effect etc ~~can be exp~~ shows that light has particle nature - ie in terms of quantum theory of light], therefore from symmetry, deBroglie ~~suggested~~ predicted that matter must also posses dual nature. Thus particles like $\bar{e}$, proton, ~~e~~ neutrons etc should also exhibit wave nature.

The waves associated with material particles in motion are called matter waves or debroglie waves and their wavelength is called de Broglie wavelength.

DeBroglie Equation

Energy of a photon of frequency ν, according to Planck's Quantum theory,

$$E = h\nu \quad \text{— (1)}$$

Considering the mass-energy equivalence of Einstein,

$$E = mc^2 \quad \text{— (2)}$$

(1) = (2) gives

$$h\nu = mc^2$$

$$\frac{hc}{\lambda} = mc^2$$

$$\boxed{\lambda = \frac{h}{mc} = \frac{h}{p}}$$

p → momentum of the particle

λ → wavelength of radiation

Acc. to De Broglie Hypothesis, it must be true for material particles of mass m moving with velocity v

$$\therefore \boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

This is called DeBroglie wave equation.

(i) $\lambda \propto \frac{1}{p}$

(ii) If $v = 0$, then $\lambda \to \infty$ ie matter waves can be associated with particles in motion

(iii) It is independent of charge of particles.

(iv) They cannot be electromagnetic in nature as em waves are only associated with accelerated particles.

(*) The de-Broglie wavelength of a particle of mass m and kinetic energy K is

$$\lambda = \frac{h}{\sqrt{2mK}}$$

(*) If a particle of mass 'm' carrying charge 'q' is accelerated through potential V, then

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

[for a particle whose absolute temp is 'T' & mass 'm'

$$\lambda = \frac{h}{\sqrt{2mk_B T}}$$

T → temp t

k_B → Boltzmann constant]

(*) De Broglie wavelength associated with the orbital e^-.

In n^{th} orbit

$$L = \frac{nh}{2\pi}$$

$$\lambda = \frac{h}{mv} = \frac{h\, 2\pi r}{nh} = \frac{2\pi r}{n}$$

$L = mvr$

$mvr = \frac{nh}{2\pi}$

$mv = \frac{nh}{2\pi r}$

No. of waves in n^{th} orbit is

$$N = \frac{2\pi r}{\lambda} = n$$

ie no. of de Broglie waves in n^{th} orbit is n.

The de Broglie wavelength associated with orbital e^- in n^{th} orbit of Hydrogen atom is

$$\lambda_n = \frac{12.27}{\sqrt{V}}$$

$$\lambda_n = \frac{12.27}{\sqrt{13.6/n^2}} = \frac{12.27\, n}{\sqrt{13.6}}$$

$$\lambda_n = 3.3n \ \mathring{A}$$

(5)

Broglie wavelength of an e^-

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

where $e \rightarrow$ charge on an e^-

$V \rightarrow$ accaluating potential

$h \rightarrow$ planck's const.

$m \rightarrow$ mass of an e^-

Subs. for e, h, m

$$\lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}} m = \frac{12.27}{\sqrt{V}} \mathring{A}.$$

* Electron microscope is an impt application of deBroglie waves designed to study very minute objects like viruses, microbes and crystal structure of solids.
* Principle of Electron microscope - like light radiations e^- beams behave as waves but with much smaller wavelengths.
* By using electric and magnetic fields, $\bar{e}$ beams can be focussed just as light beams are focussed by glass lenses.
* As the magnifying power of a microscope is inversely proportional to the wavelength of radiation used, magnifying power of $\bar{e}$ microscope can be adjusted as $\left[\lambda = \frac{12.3}{\sqrt{V}} \mathring{A}\right]$ value of accelerating potential can be selected suitably to get λ as small as possible while in

88.6
4430
40
43
40
30
90 +
95
95 886.36
93
373
373 +
70
4.43

in the case of light operated microscopes there is limit on wavelength of light.

Expt – to study the wave